M10 Honours: Section 2.4 Deriving Quadratic Equations

1. Derive a quadratic equation for each parabola with the given information:

a) Vertex $(0,2)$ with point P	b) Vertex $(5,2)$ with point P $(3,-10)$	c) Vertex $(4,-3)$ with point P $(6,5)$
(-3,11)		
d)Vertex $(0,2)$ opens down, and is	e) Vertex $(0,2)$ opens up, and is	f) X-intercepts at 3 & -4, opens down,
	congruent to $y = 0.25x^2$	and is congruent to $y = \frac{1}{2}x^2$
congruent to $y = \frac{1}{3}x^2$		
g) Roots at 3&–6 with point P	h) Roots at 7 e 3 with point P	i) Roots at 10.6. 5 with point P
(2,12) on the parabola	h) Roots at $-\frac{7}{4} & -\frac{3}{2}$ with point P	i) Roots at $10 \& -\frac{5}{6}$ with point P
	(3,8) on the parabola	(5,10) on the parabola

j) 3 Points on the parabola: $(2,6)$, $(4,6)$, and $(5,12)$	k) Maximum at 3, y-intercept at -12, and point $\left(10,-12\right)$	I) Axis of Symmetry $x=3$, y-intercept at 4.5, one x-intercept at 7, opens down

2. Determine the value of the Discriminant and the Nature of the Roots:

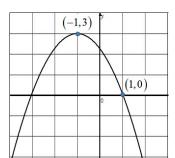
a) $4x^2 + 10x + 9 = 0$	b) $-x^2 + 6x + 7 = 0$	c) $-3x^2 + \frac{1}{2}x + 4 = 0$
		2
d) $5x^2 - 3x + \frac{1}{4} = 0$	e) $(x+3)^2 = 1$	$f)\frac{x^2}{-3} = 4x$

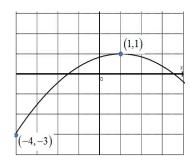
g) $200 + 33x + x^2 = 0$	g) 2	00 + 1	33 <i>x</i> +	$-x^2$	=0
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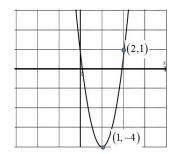
h)
$$0 = x^2 + 12x - 85$$

i)
$$0 = 3x^2 - 12x - 288$$

3. Derive a quadratic equation in the form of $y = a(x-p)^2 + q$ for each of the following graphs:







4. Determine the value(s) of "k" in each equation so that one root is double the other root:

a)
$$x^2 + kx + 50 = 0$$

b)
$$4x^2 + kx + 4 = 0$$

c)
$$2x^2 - 3x + k = 0$$

5. Determine the value(s) of "k" in each equation so that one root is triple the other:

a)
$$3x^2 - 4x + k = 0$$

b)
$$4x^2 + kx + 27 = 0$$

c)
$$16x^2 + kx + 27 = 0$$

6. Determine the value(s) of "k" in each equation so that one root is equal to the other:

a)
$$4x^2 + kx + 25 = 0$$

b)
$$9x^2 - 42x + k = 0$$
 c) $9x^2 - kx + 1 = 0$

c)
$$9x^2 - kx + 1 = 0$$

7. For what values of "k" will each equation have either "2 different roots", "2 equal roots", or "no roots"

a)
$$4x^2 + kx + 6 = 0$$
 (2 Different Roots)

a)
$$4x^2 + kx + 6 = 0$$
 (2 Different Roots) b) $6x^2 - 10x + k = 0$ (2 equal roots)

c)
$$(2k+1)x^2 - 3x + 5 = 0$$
 (No Roots)

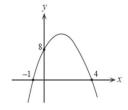
c)
$$(2k+1)x^2 - 3x + 5 = 0$$
 (No Roots) d) $(5k-1)x^2 - 4kx + 1 = 0$ (2 Distinct Roots)

e)
$$kx^2 - 5x + 2k = 0$$
 (2 Equal Roots

e)
$$kx^2 - 5x + 2k = 0$$
 (2 Equal Roots) f) $(2k-1)x^2 + 3kx + k + 1 = 0$ (2 Distinct Roots)

8. f(x) is a quadratic function with zeroes at x = 2 and x = 4. f(0) = 1, what is f(-1)?

9. In the diagram, the parabola has x-intercepts at -1 and 4, with the y-intercept at 8. If the parabola passes through the point (3, w), what is the value of "w"?



10. Let m and n be the roots of the equation $ax^2 + bx + c = 0$. Let $px^2 + qx + r = 0$ be a quadratic equation for which m+2 and n+2 are roots. If p=a , then q+r , expressed in terms of a , b , and c is:

a)
$$c+3b$$

b)
$$c-h$$

b)
$$c-b$$
 c) $c+3b+8a$ d) $c-b+4a$ e) $c-b+8a$

d)
$$c - b + 4c$$

e)
$$c - b + 8a$$

- 11. Both roots of the quadratic equation $x^2 63x + k = 0$ are prime numbers. What is the number of possible values for the constant "k":
 - a) 0

- b) 1
- c) 2

d) 4

- e) more than four
- 12. A grid point in the place is a point (x, y) for which both x and y are integers. The number of grid points that lie within or on the boundary of the region bounded by the parabola $y = x^2$ and the line y = 50 is:
 - a) 470
- b) 485
- c) 490
- d) 750
- e) 765

13. What value(s) of k will make the following to be factored as the square of a linear polynomial? $4x^2 + kx + 49$

- 14. For what values of k does the equation $5x^2 + kx + 5 = 0$ have two different real roots?
 - a) -10 < k < 10
- b) k < -10, k > 10
- c) $k \pm 10$
- d) k > 10

		2				
15.	One root of a	$x^2 - kx +$	18 = 0	is twice the	other. Then	k=2

- 16. The quadratic equation $x^2 + mx + n = 0$ has roots that are twice those of $x^2 + px + m = 0$, and none of m, n, and p is zero. What is the value of n/p?
 - a) 1

- b) 2
- c) 4

- d) 8
- e) 16

- 17. For how many integers k do the parabolas with equations $y = -\frac{1}{8}x^2 + 4$ and $y = x^2 k$ intersect on or above the x-axis?
 - a) 9

- b) 32
- c) 33

- d) 36
- e) 37

18. Determine the one value of "x" such that $x - \sqrt{4x + 12} = 0$. Justify your answer

19. Challenge: COMC 2015: A quadratic function in the form of $y = ax^2 + px + q$ has one root that is double the other.

i)	If $p = -$	-15	. then	what	is the	value	of "a"?

ii) If one of the root is equal to 4, then what are all the possible values of
$$p+q$$
?

iii) If the value of
$$p+q=9$$
, then what are all the possible quadratic equations?

20. Challenge: Determine all real values of "c" such that
$$x^2 - 4x - c - \sqrt{8x^2 - 32x - 8c} = 0$$
 has precisely two distinct real solutions for "x". [COMC]

21. Let "x", "y", and "z" be real numbers such that
$$x + y + z = 3$$
 and $xy + yz + xz = a$, where "a" is a real parameter. Determine the value of the parameter "a" for which the difference between the maximum and the minimum possible values of "x" equals 8. (COMC POW)